# Chapter 4

## Production and Labor

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# The Real Economy

- We now begin our dive into macroeconomic modeling
  - Economists use complex models as it's generally hard (and usually unethical) to run nation-wide experiments

- The first model we'll build is a *Real* Business Cycle model
  - But first, we need to define its parts

# Partial Equilibrium

- **Partial equilibrium (PE)** *not* all markets are in equilibrium
- **General equilibrium (GE)** all markets are in equilibrium

- We'll study PE in individual markets first, then move to GE
  1. Labor market
  2. Goods/loanable funds market(s)
  3. Money/asset market(s) – post exam 1

# Partial Equilibrium in the Labor Market

1. Discuss basics of production
2. Microfoundations of labor demand & profit maximization
3. Microfoundations of labor supply & utility maximization
4. Market clearing (supply + demand)
5. Aggregate summary

› Sets up how we'll study goods & money markets later

# Factors of Production

- **Factors of production** are resources used to make goods and services

- Inputs to production that make outputs:
  - Labor (*N*)
  - Capital (*K*)
  - Other: land, financing, energy, entrepreneurship

# Relationship b/t Inputs and Output

- One way to describe the relationship between inputs and output is with a function:

$$Y = zF(K,N)$$

- Where: $Y$ = output

  $z$ = "total factor productivity, TFP" (technology)

  $K$ = capital

  $N$ = labor

# Functional Form

› Usually, we assume *F()* is a Cobb–Douglas function

$$Y = zK^{\alpha}N^{v}$$

› Where: $\alpha$ = capital share of output

$v$ = labor share of output

› Easy to find total *Y, K, N* and $\alpha$, $v$ from data

– We can then back out TFP

# Cobb–Douglas and Returns to Scale

- **Returns to scale:** how much more output do we get when we increase an input?

- Increasing returns to scale:
  - Double *all* inputs, get more than double the output

- Constant returns to scale:
  - Double *all* inputs, get exactly double the output

- Decreasing returns to scale:
  - Double *all* inputs, get less than double the output

# Identifying Returns to Scale

- Look at sum of exponents:
  - $\alpha + v > 1$ ➔ increasing returns
  - $\alpha + v = 1$ ➔ constant returns
  - $\alpha + v < 1$ ➔ decreasing returns

- Typical U.S. firm $\alpha$ and $v$ estimates are 0.27 and 0.62
  - Implying DRS at the firm-level
  - Aggregate economy is (roughly) CRS

# The Representative Agent

- For now: only 1 firm that represents all firms in our model
  - This is the representative firm (or household)
  - Further assume 1 good produced (thus only 1 price)

- Usually for quantities, capital letters denote aggregates; lower case letters denote firm/household level variables
  - But since there is only 1 representative firm, $K = k$
    - I would love to say this notation is always true, but it is the economist's job to make the obvious opaque, so this may change

# Profit Maximization: Labor and Capital

- Firms pick labor and capital for profit maximization
  - Ch. 2 assumption: firms only care about maximizing value
  - Remember: firms demand labor; HHs supply labor

***Optimal quantity: marginal benefit = marginal cost***

# Marginal Productivity

- To figure out the marginal benefit of inputs, we need their marginal productivities
  - Productivity of *k* and *n* are the benefits to firm value
  - This chapter will focus on labor, but the general approach applies for capital

- **Marginal Productivity of Labor (*MPN*):**
- Change in output from adding 1 more unit of labor

- **Marginal Productivity of Capital (*MPK*):**
- Change in output from adding 1 more unit of capital

# Marginal Productivity of Labor

› Change in output from adding 1 more unit of labor:

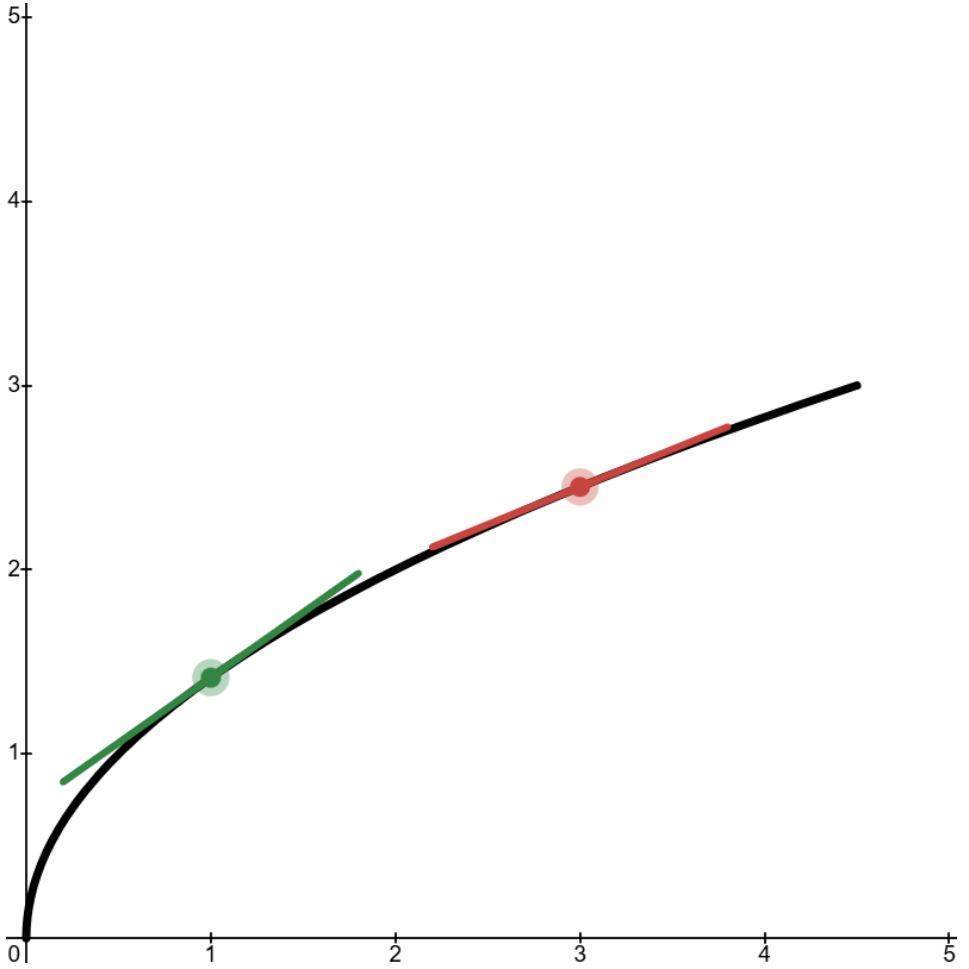
$$\frac{\Delta Y}{\Delta N}$$ , where Δ denotes change

› Alternatively, one can use calculus to find:

$$\frac{dY}{dN}$$

# Slope

- These equations give us the slope at a point
  - Rate of change at a point
  - How Y changes with N
- Notice the slope gets flatter as the x-value rises



# Properties of Marginal Product of Labor

1. *MPN* is positive: Y increases with more labor

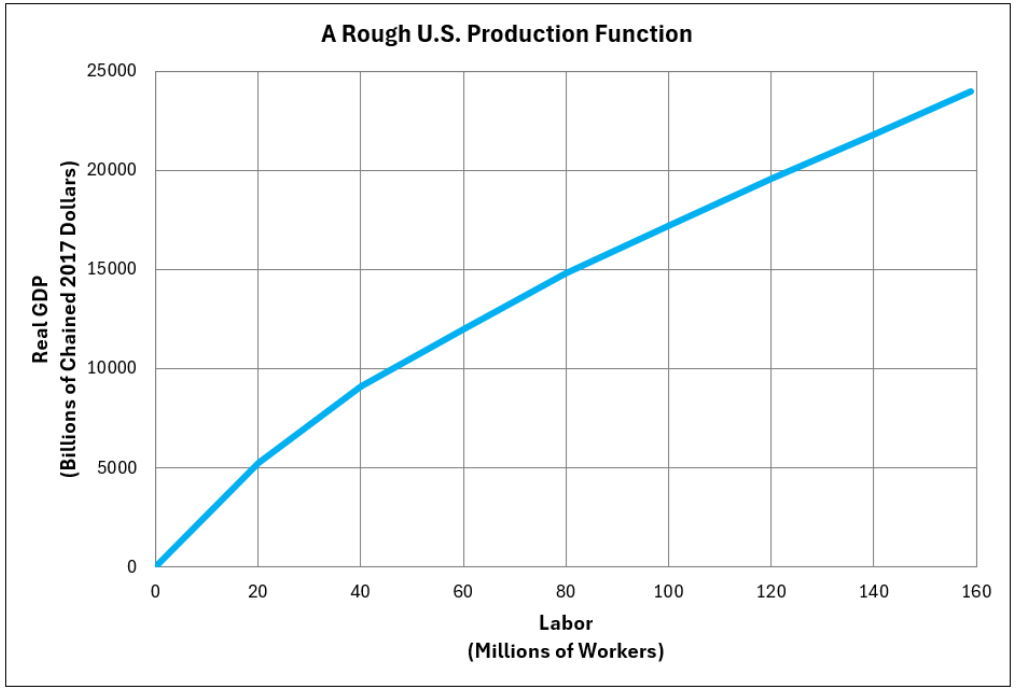
2. But *MPN* decreases as *N* increases

› **Law of diminishing returns:**
  - *Additional* output falls as inputs increase
  - You get a lot of homework done with 1 computer
  - Not much additional homework done with 10 computers
  - "Too many chefs spoil the stew"

› Applies to *K*, as well!

# Diminishing Production on a Graph

- As *N* rises, *Y* rises
  - More capital = more output
- But it starts to flatten out in the limit
  - Can't grow forever
    - Yes, technically, you still grow some epsilon (arbitrarily small) amount




Data: All Employees, Total Nonfarm (BLS, 2026) and Real Gross Domestic Product (BEA, 2026)

# The Wage

- The last ingredient is the marginal cost
- The [nominal] **Wage (*W*)** is the price of a unit of labor
  - For simplicity we'll assume no worker benefits
- The **Real Wage (*w*)** is the nominal wage in terms of the output good $\left(\frac{W}{P}\right)$
  - Purchasing power of wages
- *Note: for prices, capital letters denote nominal variables; lower case letters denote real variables*

# Profit Maximization thru Labor

› Firm's labor demand for profit maximization:

$$MPN = w$$

› This ensures profit maximization for the firm
  – Also called the Firm "optimality" or "market clearing" condition

# Profit Maximization Example (1/3)

› Assume the economy produces a consumption good, ***c***, and the real wage is 2.00

| Y | N | MPN | w | Profit |
|---|---|---|---|---|
| 0 | 0 | - | 2 | 0 |
| 7 | 1 | | 2 | |
| 11 | 2 | | 2 | |
| 13 | 3 | | 2 | |
| 14 | 4 | | 2 | |
| 14 | 5 | | 2 | |

# Profit Maximization (2/3) – Filling out *MPN*

| Y | N | MPN | w | Profit |
|---|---|---|---|---|
| 0 | 0 | - | 2 | 0 |
| 7 | 1 | 7 | 2 | |
| 11 | 2 | 4 | 2 | |
| 13 | 3 | 2 | 2 | |
| 14 | 4 | 1 | 2 | |
| 14 | 5 | 0 | 2 | |

# Profit Maximization (3/3) – Finding Profit

| Y | N | MPN | w | Profit |
|---|---|---|---|---|
| 0 | 0 | - | 2 | 0 |
| 7 | 1 | 7 | 2 | 5 |
| 11 | 2 | 4 | 2 | 7 |
| 13 | 3 | 2 | 2 | 7 |
| 14 | 4 | 1 | 2 | 6 |
| 14 | 5 | 0 | 2 | 4 |

Real Profits = $Y - w \times N$

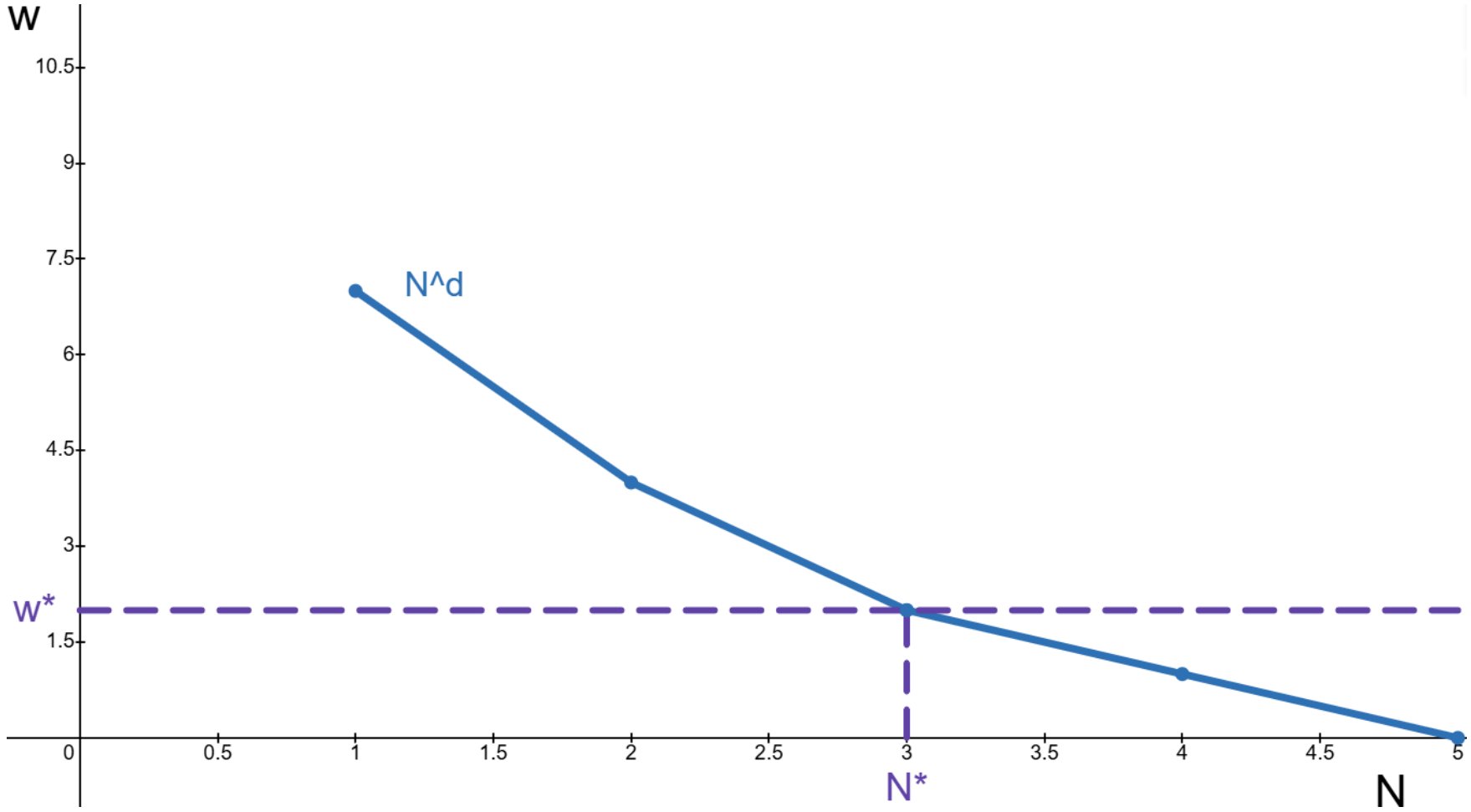
- The same analysis works for the nominal wage, too (multiply by $P$).

# A Note on 1-good Economies

- We study a 1 good economy for simplicity
  - Or a variable that represents the average basket of goods

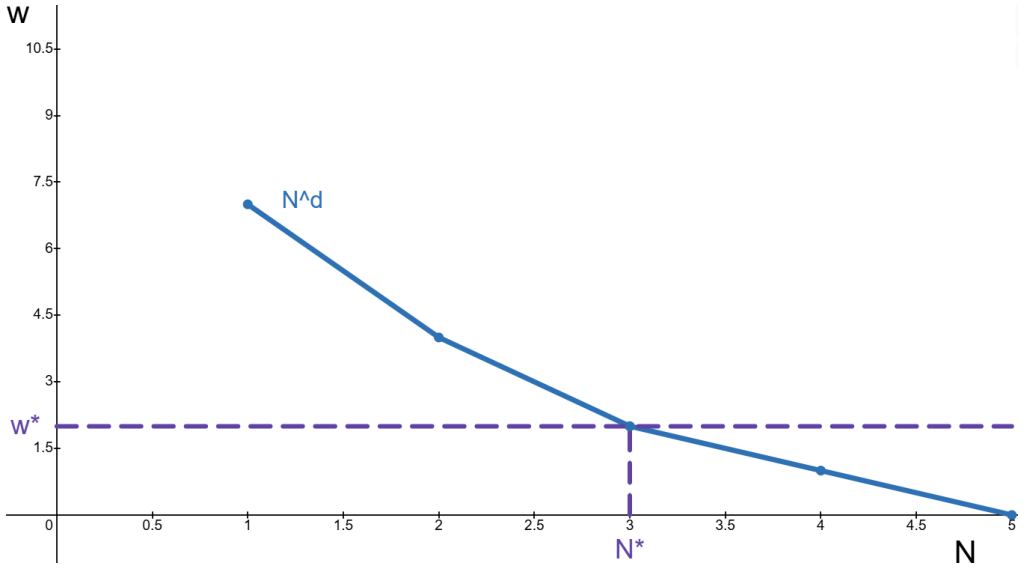
- 1 output good → 1 output price
  - Don't need CPI, PCE to determine price level
  - The price level is the 1 good's price

# Graphing Total Labor Demand



# Interpreting the Graph

- The points on the curve are the *MPN* values
  - $N^d = NPN$
- If *all* labor becomes more productive, *MPN* ↑
  - $N^d$ ↑
- If the firm wants to lower *MPN*, it decreases *N*



# The Supply of Labor

- The supply of labor comes from households

- Two starting assumptions for households:
  - HHs like **leisure ($\ell$)**, time not spent working
    - Equivalent to saying they dislike labor
  - They like consumption ($c$), but need labor income to afford it

# Time Endowment Constraint

› The HH is endowed with some amount of time each day
  - For simplicity, normalize to 1
  - They have 100% of their day to divide into labor or leisure

› Describe $\ell$ and $n^s$ in terms of *fraction of hours* in the day:

$$1 = \ell + n^s$$

  - Easier to substitute away a variable to shrink problem
  - HH gets utility from leisure, not labor

# Extensive and Intensive Labor Choice

- Two choices for labor: extensive and intensive

- **Extensive labor choice:** Do you work? Yes or no
  - Only 1 of 2 answers
- **Intensive labor choice:** How much do you work?
  - Continuum of answers

- We'll assume everyone is making an intensive labor choice

# The Household's Problem

› In real terms, the HH's problem is:

$$\max\{U(c, \ell)\} \text{ st: } c < w \times (1 - \ell) + X$$

- Maximize utility from $c$ and $\ell$, *subject to* their budget
- Budget is (wage × hours), plus income from other sources, $X$
  - $X$ = NFP, taxes, transfers, asset returns, etc
- HH cannot spend more than their total income

# Analyzing the Household's Budget

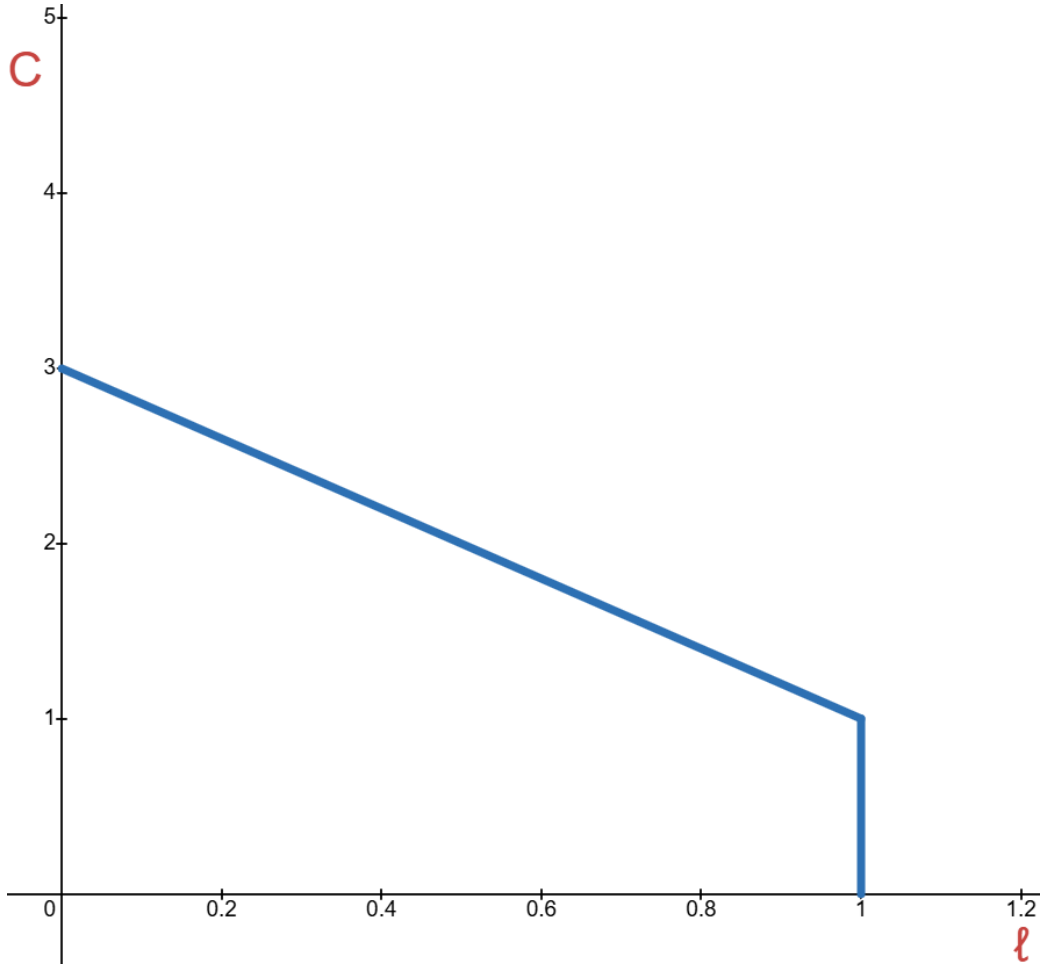
- The budget is: $c < w \times (1 - \ell) + X$
- We can assume equality: $c = w \times (1 - \ell) + X$
  - Why?
- What does this equation look like now?

# Relation to Slope-intercept Equation

- $c = w \times (1 - \ell) + X$
  - $c$: y-axis
  - $\ell$: x-axis
  - $-w$: slope
  - $X$ is tricky to pinpoint; not the y-intercept

- $X$ = amount of $c$ HH can afford without working
  - Consider when $\ell = 1$

# Graphing HH Budget



# Labor-Leisure Trade-off

- What does the HH choose?
  - Again, want marginal benefit = marginal cost

- Marginal benefit of leisure:
  - Marginal utility of leisure: $\boldsymbol{MU(\ell)}$

- Marginal cost of leisure, in utility terms:
  - Give up some wages and potential *c*
  - Marginal utility of *affordable c* given up: $\boldsymbol{w \times MU(c)}$

# Utility Maximization thru Labor

› HH chooses leisure/supplies labor for utility maximization:

$$\boldsymbol{MU(\ell) = w \times MU(c)}$$

› Commonly written as: $\frac{MU(\ell)}{MU(c)} = w$

– Also called the HH "optimality" or "market clearing" condition

› Like *MPN* and *MPK*, we assume *MU* **diminishes**

# Marginal Rate of Substitution

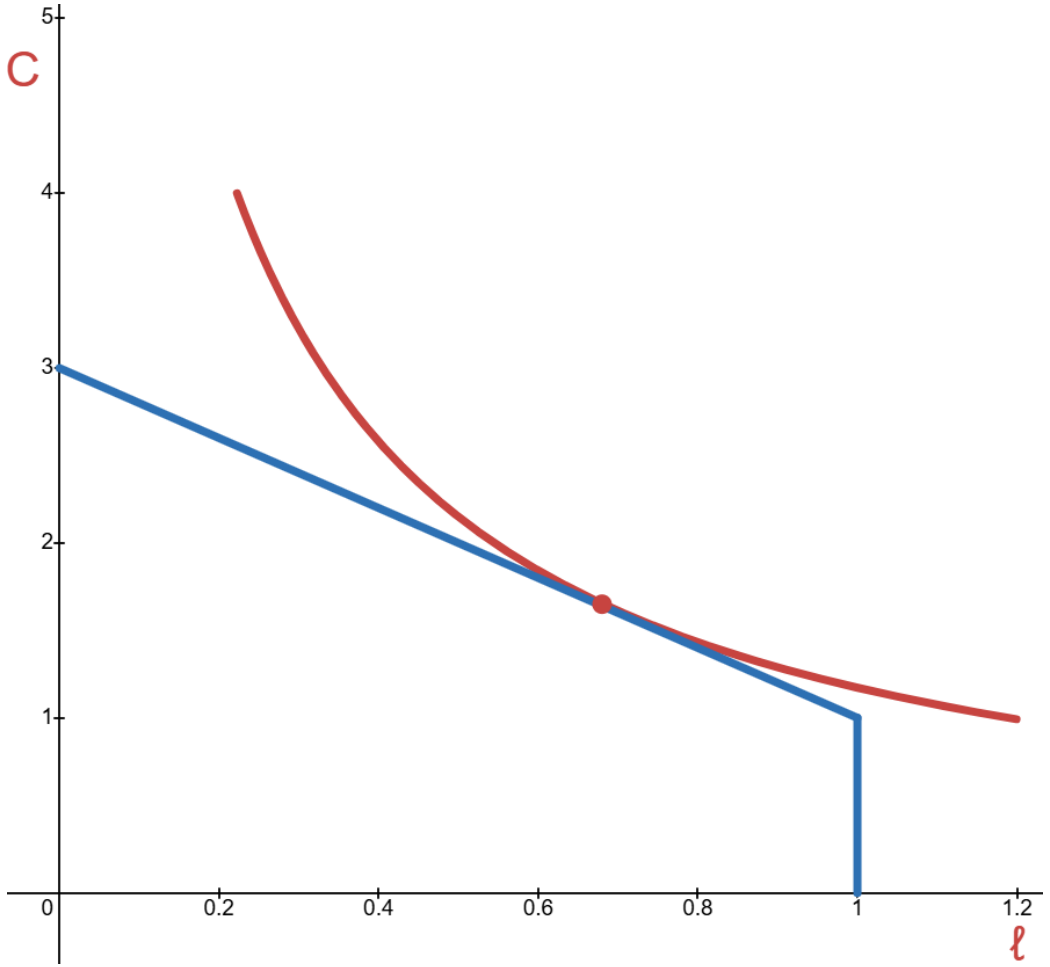
- **Marginal Rate of Substitution ($MRS_{x,y}$)** defines the rate at how a consumer will trade good x for good y
  - We know the two "goods" are $\ell$ and $c$

- From the last equation:

$$MRS_{\ell,c} = \frac{\text{MU}(\ell)}{\text{MU(c)}} = w$$

# Proof (Not required)

# Graphing HH's Choice



# Relative Price

- The real wage is the **relative price** of leisure
  - Price of one good in terms of another
  - By consuming more leisure, you give up the consumption you could have had with that labor income

$$\frac{MU(\ell)}{MU(c)} = \boldsymbol{w}$$

  - Real wage must afford enough consumption to make up for the utility lost by giving up leisure
  - HH is indifferent between extra unit of $c$ or $\ell$

# Real Wage and Intensive Labor Choice

- An *increase* in the real wage can affect this in two ways:
  - Income effect*
  - Substitution effect

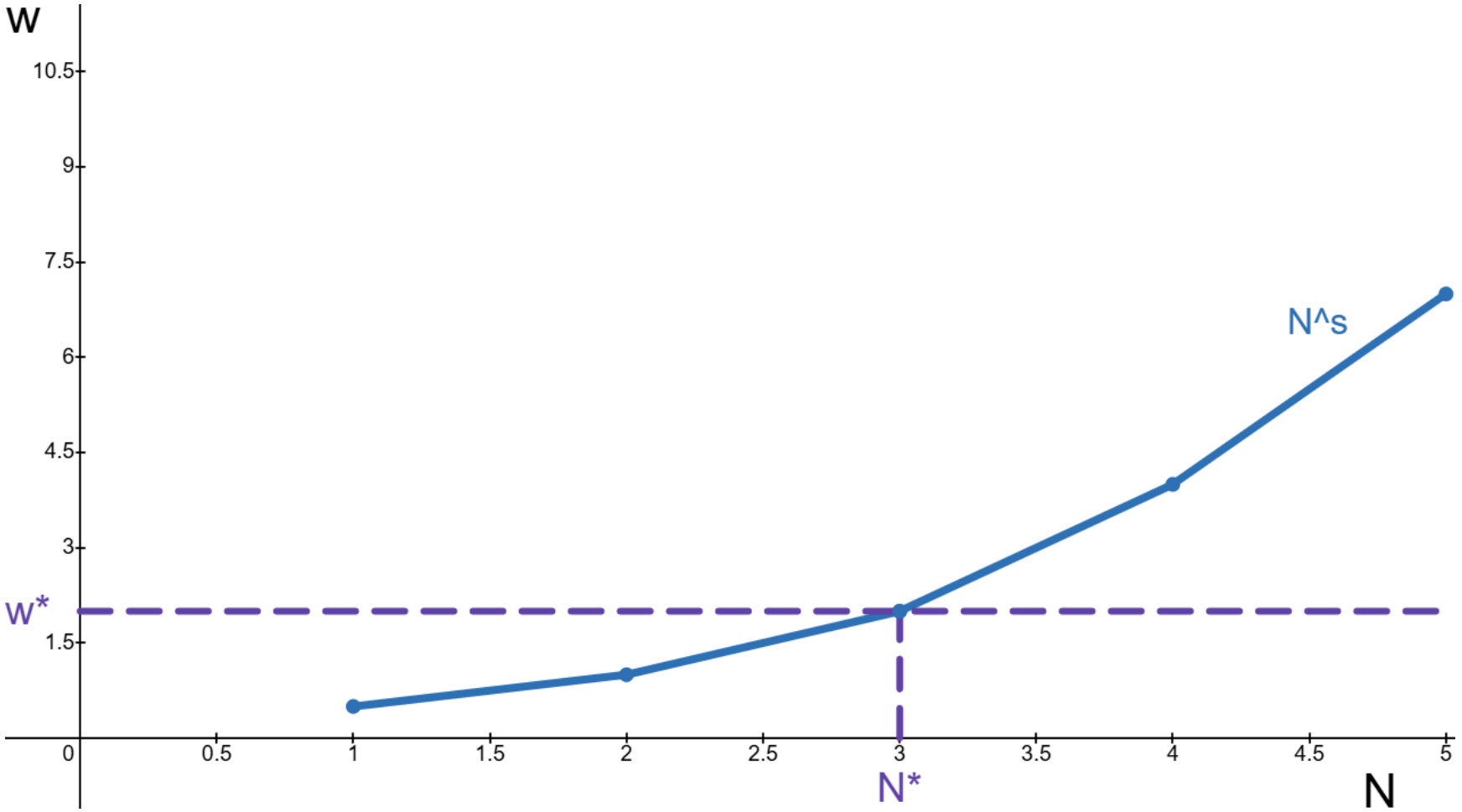
- **Income effect:** you are richer; you consume more goods
  - Leisure is treated as a good; *you work less*
- **Substitution effect:** relative price of leisure is higher
  - Substitute away from expensive good; y*ou work more*

- *Recall through our lens, income increases wealth. However, labeling this as a wealth effect is awkward.

# Substitution > Income

- In general, we assume the substitution effect dominates
  - Increase in the real wage makes people work more
  - Labor supply curve slopes up
    - At very high wage or wealth levels, the income effect may dominate. Why?

# Graphing Real Labor Supply



# Marginal Rate of Transformation

- Before we move to equilibrium, we have to define one more rate

- **Marginal Rate of Transformation ($MRT_{x,y}$)** defines the rate at how one good is transformed into another
  - We know the two "goods" are $\ell$ and $c$

- From the firm's condition:

$$MRT_{\ell,c} = MPN = w$$

# HH and Firm Condition(s)

$$\frac{\text{MU}(\ell)}{\text{MU}(c)} = MPN = w$$

$$MRS_{\ell,c} = MRT_{\ell,c} = w$$

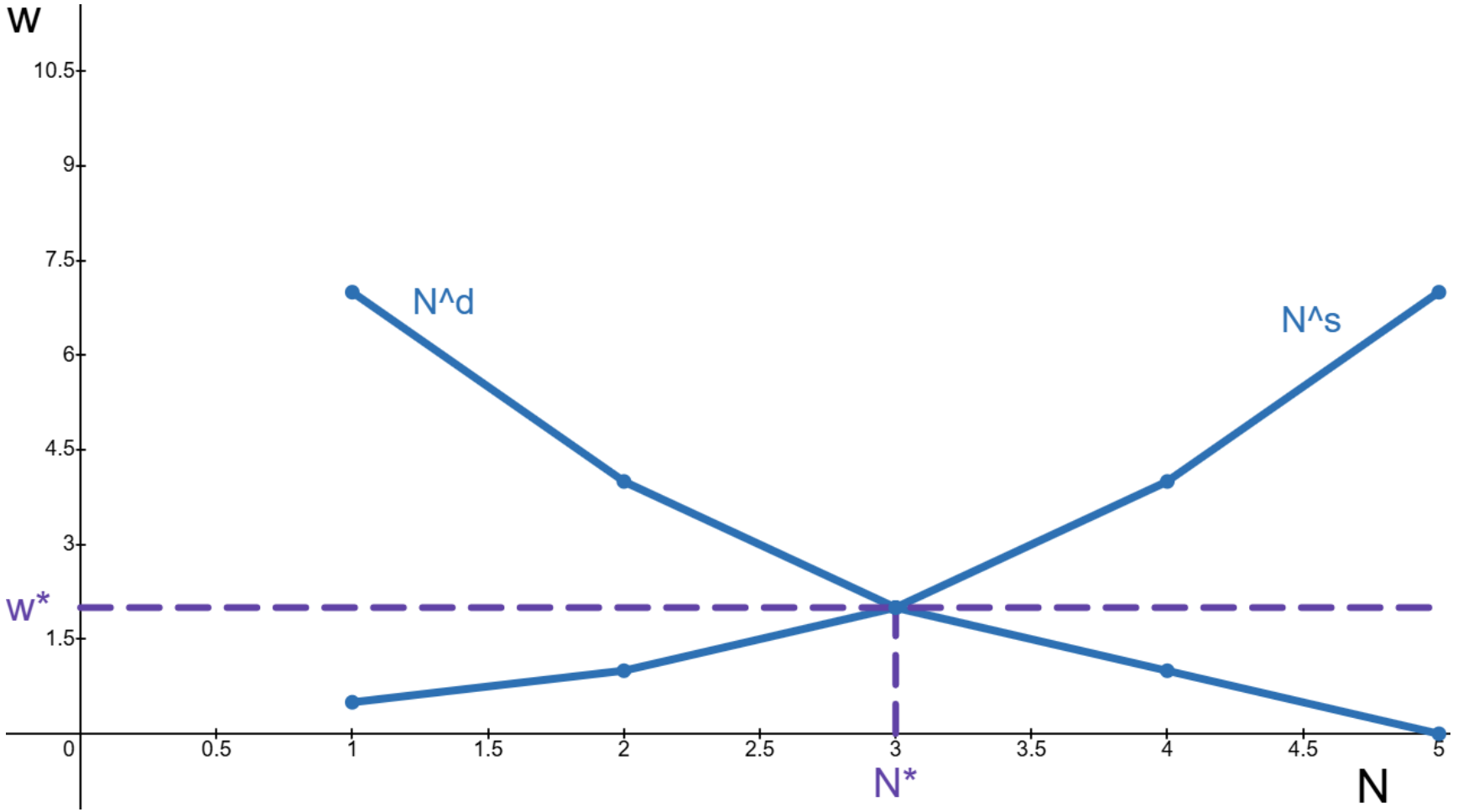
› This condition ensures labor market clearing

# Market Clearing, again

- **Market clearing:** price(s), demand function(s), and supply function(s) where quantity supplied = quantity demanded
  - Here, the quantity is labor and the price is the real wage
  - ❖ Real wage where quantity labor supplied = quantity labor manded

- Our example: **market clearing wage = 2**
  - Labor supplied = labor demanded

# Graphing Market Clearing



# Changes in the Labor Market

- Markets are **dynamic**; things can change

- We can have **shocks** to supply, demand, and price
  - These shocks can also be dynamic, or permanent

# Demand Shocks

- A **demand shock** is anything that changes demand (for labor) at a given price (of labor)
  - Anything that changes the *MPN*

- **Positive demand shocks (shift demand right)**

1. Better technology ($z$)
2. More capital ($k$)
   - What does this assume about capital and labor?

# Example: Increase in *k*

› An increase in *k* increases labor demand

# Supply Shocks

- A **supply shock** is anything that changes supply (of labor) at a given price (of labor)

- **Positive supply shocks (shift supply right)**

1. Increase in labor force
   - Raise social security age, more immigration, etc
2. Lower wealth
   - Can't consume as much so MU(*c*) increases
3. Lower *future* wages
   - Similar reasons to #2. Note on this later

# Example: Fall in Future Wages

› Fall in future wages increases labor supply

# The Y-axis Does NOT Shift Curves

- Remember: whatever variable is on the y-axis, in this chapter it's the real wage (*w*), will NEVER cause a shift
- *What we've been looking at is the current real wage (w)*
- A change in the future wage (*w'*) is not the y-axis variable and will therefore cause a shift
  - Technical terms: *w'* is not the argument to the $N^s$ function

# The Labor Force (BLS, 2026)

- **Labor force:** 16+ yrs old, noninstitutional civilians
  - Employed
  - Unemployed

- **Labor force participation rate** is roughly 63%
  - (labor force ÷ available population)*100

- **Unemployed:** actively looking for work in the last 4 weeks

- **Unemployment rate** is roughly 5%
  - (total unemployed ÷ total labor force)*100

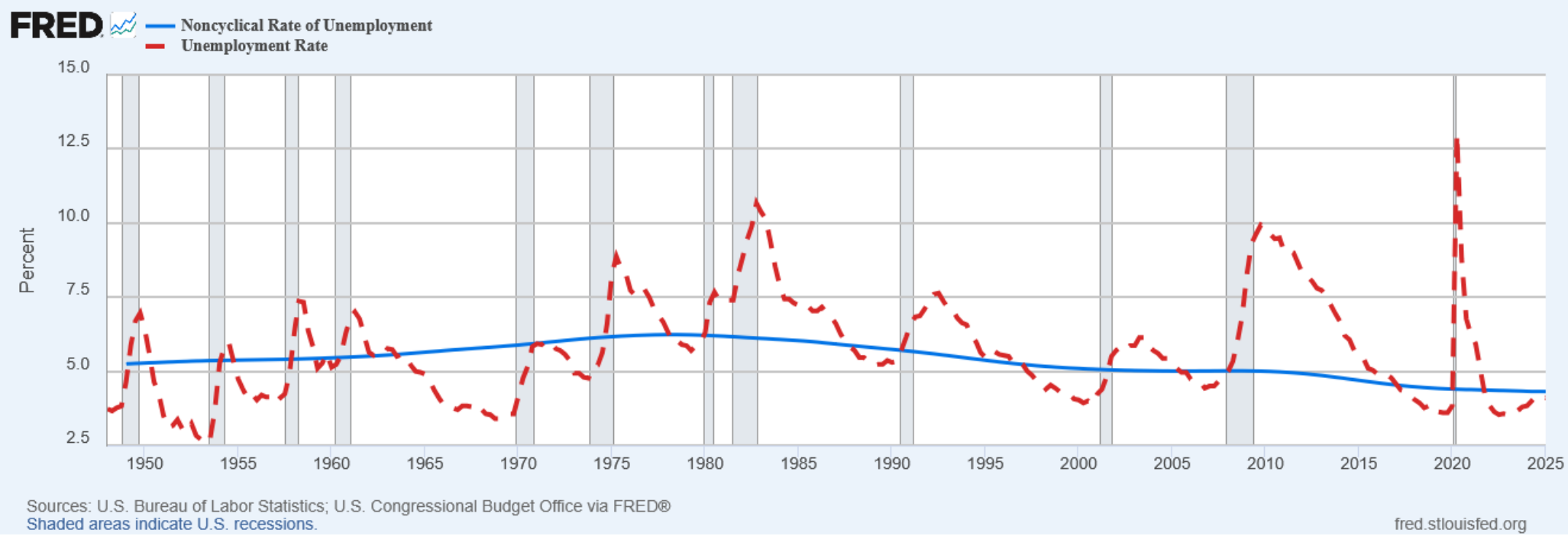
# Types of Unemployment

- Frictional unemployment
  - Takes time to find a job
  - Usually, people looking for something that suits them
- Structural unemployment
  - Skill mismatch: no one is hiring a VCR repair worker
  - Training them to be semiconductor manufactures takes time
- Cyclical unemployment
  - Business cycles; unemployment is higher in recessions

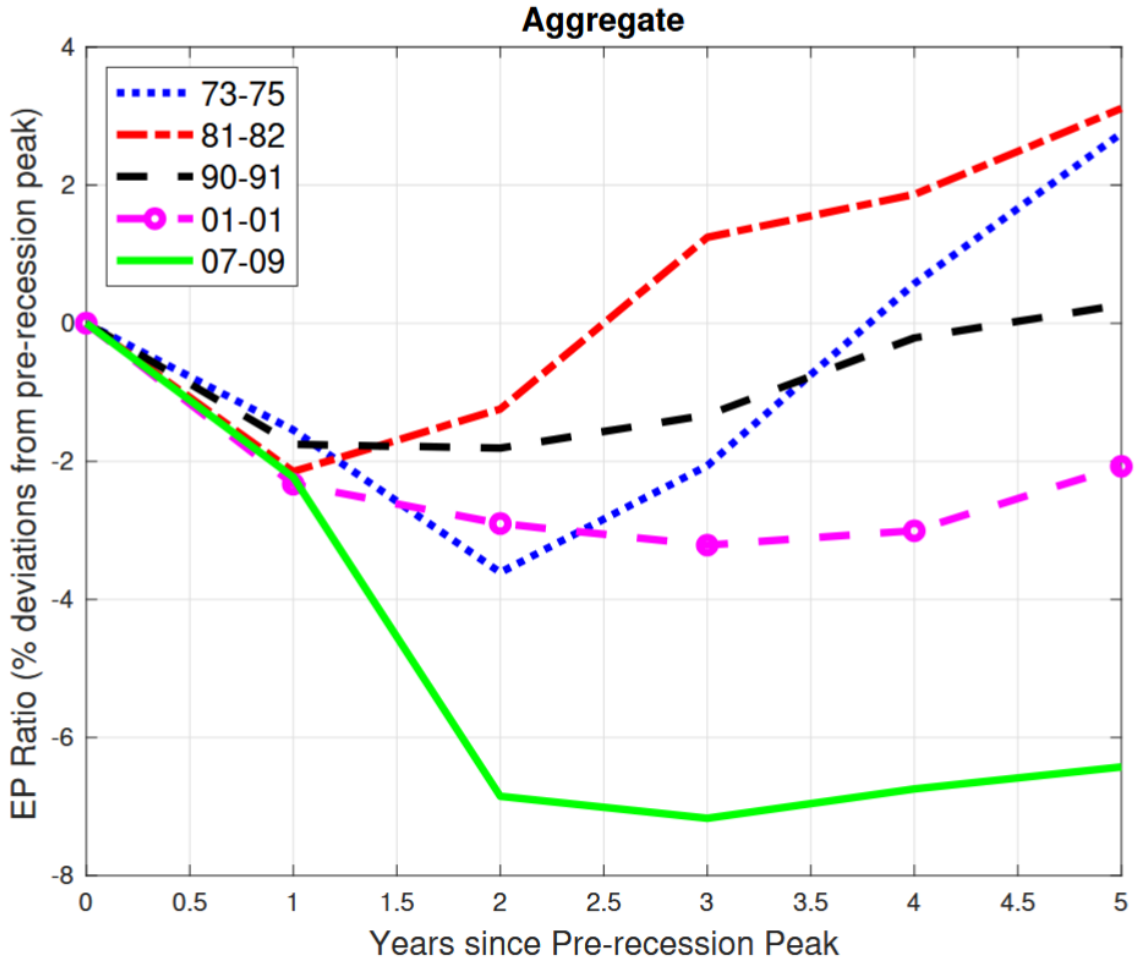
# Noncyclical Rate of Unemployment

- **Noncyclical Rate of Unemployment:**
  - Unemployment when cyclical rate = 0
  - Can still have frictional & structural unemployment
  - Used to be called the natural rate of unemployment
  - Economy is said to be at **Full Employment**

# Data: Noncyclical Rate of Unemployment

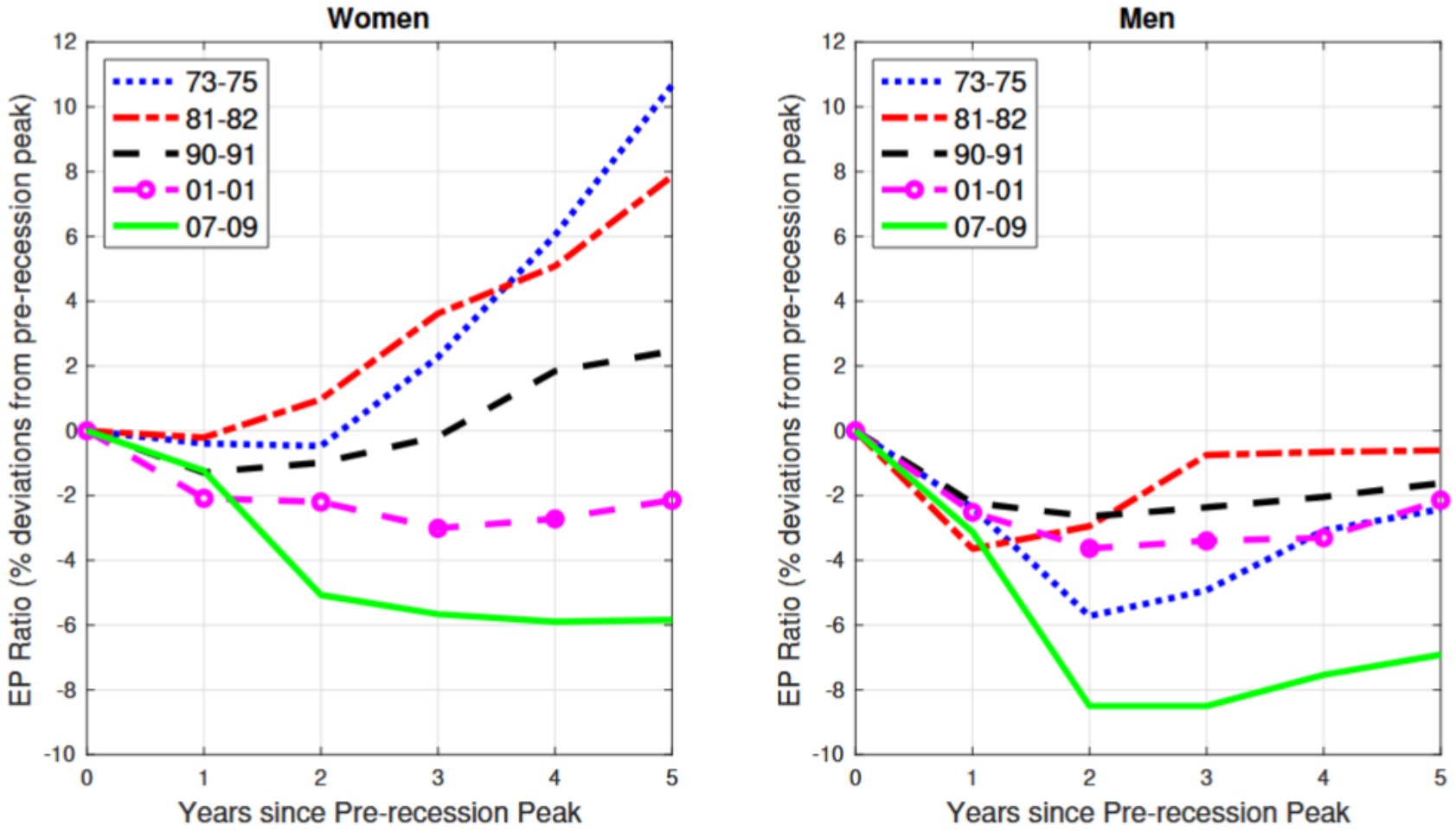


# Jobless Recoveries?




(Chakraborty, 2022. Figure B3a)

# Chakraborty (2022)




(Chakraborty, 2022. Figure B3b)

# References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2. Card, David, and Alan B. Krueger. "Minimum wages and employment: A case study of the fast food industry in New Jersey and Pennsylvania." *The American Economic Review*, Vol. 84, No. 4. (Sep., 1994), pp. 772-793.

3. Chakraborty, Pubali. "Female Labor Supply and Jobless Recovery." (2022). Working Paper.

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7. U.S. Bureau of Labor Statistics, Labor Force Participation Rate [CIVPART], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CIVPART, April 30, 2026.

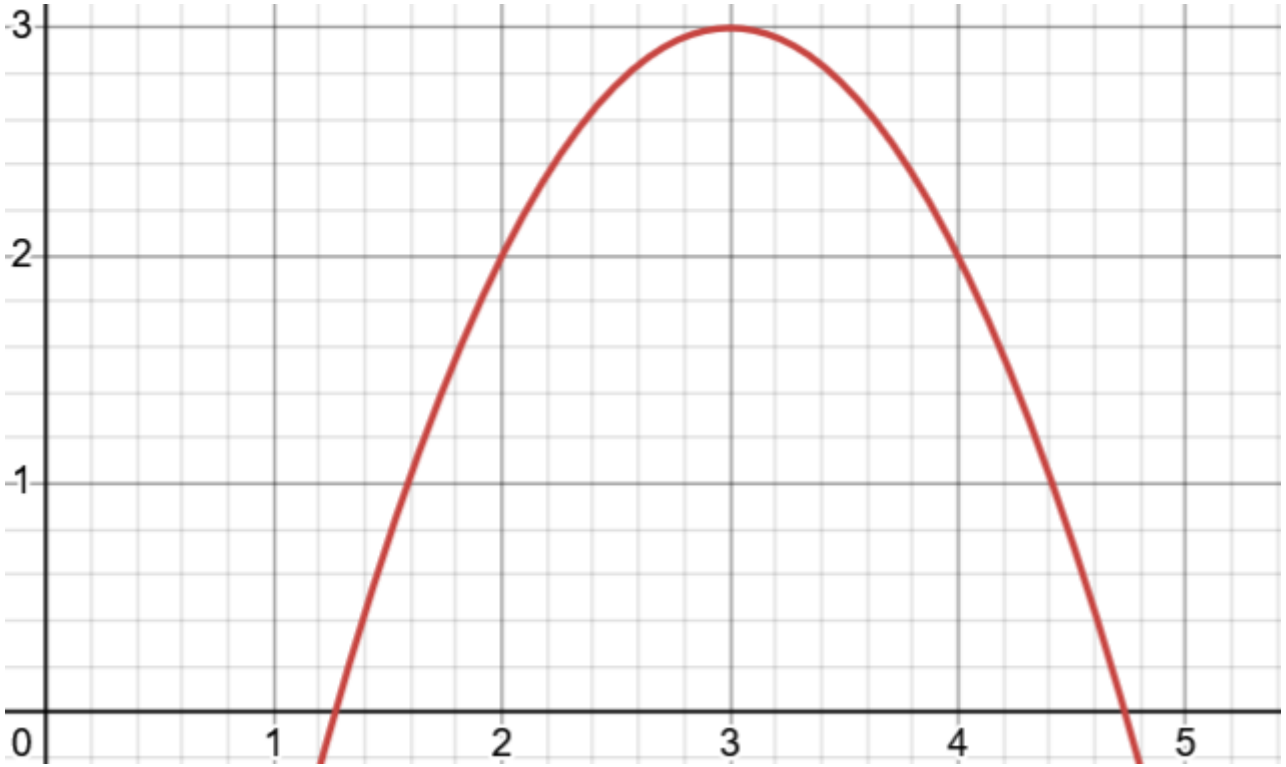
8. U.S. Bureau of Labor Statistics, Unemployment Rate [UNRATE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE, .

9. U.S. Congressional Budget Office, Noncyclical Rate of Unemployment [NROU], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/NROU, .

A. All non-cited graphs for this course made by me on Desmos.

# APPENDIX

# Why Two Equal Proft Values on Slide 22?

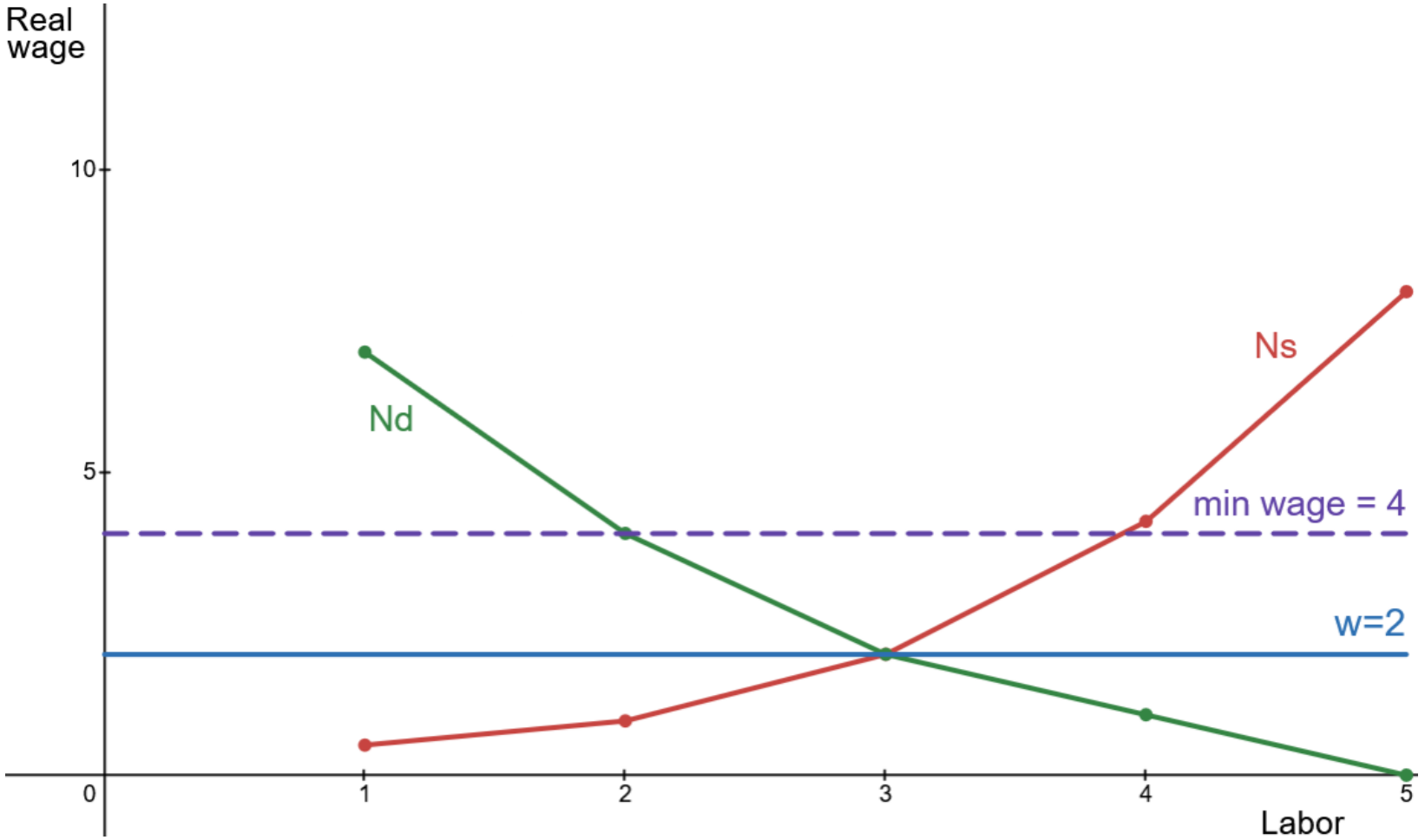


# Minimum Wage

- **Minimum wage** sets the lowest *legal* wage in the market
  - A change in the minimum wage can be a price shock

- **Binding minimum wage** forces a change in the market
  - Leads to excess supply of labor; more people want to work than the market demands
  - In our example, any minimum (real) wage over 2 is binding
  - A minimum wage of 4 leads to and excess supply of roughly 2

# Binding Minimum Wage



# Minimum Wages and Employment (Card & Krueger, 1994)

- NJ min wage went from $4.25 to $5.05 while PA had no change
- Compare fast food employment on the border of the two states
  - Why on the border?
  - No significant changes **(Card & Krueger, 1994)**
- So, a higher minimum wage does not lead to unemployment?
- Well, yes and no
  - Many micro-level firm dynamics at play (two-year warning period)
  - Cannot say that larger increases will not cause unemployment
  - Prices of fast food went up to cover costs
  - Conflicts with other studies, no long-term analysis (employment or new firms)

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