Firm Debt Relief in Financial Downturn

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Can targeting debt relief improve stabilization policy in a financial crisis?

Firm debt relief is more common; policies vary on subsets of firms targeted:

- Paycheck Protection Program (2020-2021): Smaller firms
- Auto industry bailouts (2008-2009): "Big 3" in US auto

Differing economic support:

- Smaller, younger firms rely more on debt for investment (Faff et al., 2016)
- Large firms account for greater changes in aggregates (Crouzet & Mehrotra, 2020)
- ...while generating a vast literature

Can targeting debt relief improve stabilization policy in a financial crisis?

- ▷ How should we build these targets?

Answering the Question

Non-linear general equilibrium model with firm heterogeneity and financial frictions

Heterogeneity:

- Analyze targeting different subsets of firms
- Shape of distribution matters for aggregation outcomes
  - Unconditional size distribution matching U.S. firms
  - *Untargeted* age-size distribution of U.S. firms

▷ How: Persistent idiosyncratic productivity from bounded Pareto

Financial friction:

- Collateralized borrowing $\rightarrow$ slow growth for young, small firms
- Capital misallocation

▷ Crisis: Shock to collateral constraint

Examined Debt Relief Policies

Debt relief policy where government pays a fraction of firm debt:

1. Targeting largest excess return to investment
   - ▷ Expected discounted marginal benefit of capital investment minus cost
   - Effective policy, may not be readily available to policymakers
2. Size-targeted policy
   - ▷ Small, medium, and large firms
   - Measures readily available to policymakers; historical precedents
3. Age-targeted policy
   - ▷ Young, middle-age, mature firms
   - Time permitting

Firms

Firm state (k, b, ε, a): capital, debt, idiosyncratic productivity, age

- DRS production technology: $z\varepsilon F(k, n)$
- Enter period with ε; retain with probability ρ_ε
  - Probability $(1 - \rho_\varepsilon)$ draw new ε from bounded Pareto distribution
- Age dependent exit shock: $\pi_d(a)$
  - Known before production
  - Considerations for age-based policies
- Intertemporal decisions on k' and b'
  - Collateralized debt limit: $b' \leq \zeta k$
  - Risk-free loan discount factor: $q(S)$

Aggregate state: $(\zeta, z, \mu, \theta) = S$

Firm Budget Constraint & Debt Relief

Budget:

$$D \leq x(k, b, \varepsilon, a; S) - k' + q(S)b'$$

Cash:

$$\begin{aligned} x(k, b, \varepsilon, a; S) = z\varepsilon F(k, n) - (1 + \tau(S))w(S)n(k, \varepsilon; S) + \\ (1 - \delta)k - (1 - \mathcal{J}(b)g(k, b, \varepsilon, a; S))b \end{aligned}$$

where: $\mathcal{J}(b) = \begin{cases} 1 & \text{if } b > 0 \\ 0 & \text{if } b \leq 0 \end{cases}$

Firm Problem

Start of Period Value:

$$V_0(k, b, \varepsilon, a; S_l) = \pi_d(a) \underbrace{x(k, b, \varepsilon_i, a; S_l)}_{\text{exiting value}} + (1 - \pi_d(a)) \underbrace{V(k, b, \varepsilon, a; S_l)}_{\text{continuation value}}$$

Continuation Value:

$$V(k, b, \varepsilon_i, a; S_l) = \max_{k', b', D} \left[D + \sum_{m=1}^{N_s} \pi^s_{l,m} d_m(S_l) \sum_{j=1}^{N_\varepsilon} \pi^\varepsilon_{i,j} V_0(k', b', \varepsilon_j, a'; S'_m) \right]$$

subject to:

$$0 \leq D \leq x(k, b, \varepsilon_i, a; S_l) + q(S_l)b' - k'$$
$$b' \leq \zeta k$$
$$\mu' = \Gamma(S_l)$$

Government

- Total current-period debt relief: $\int g(k, b, \varepsilon, a; S) b\mu(d[k \times b \times \varepsilon \times a])$
- Borrows: θ' at risk-free rate
- Levies payroll tax: $\tau(S)$ when paying outstanding debt obligations: θ

Budget constraint:

$$\tau(S)w(S)N(S) + q(S)\theta' \geq \theta + \int g(k, b, \varepsilon, a; S) b\mu(d[k \times b \times \varepsilon \times a])$$

Policy funded by government debt; when repayment begins, $\tau(S)$ must also satisfy fiscal rule:

$$\theta' = (1 - \phi)\theta$$

- ϕ is the fraction of public debt paid per period

Determining $\tau(S)$

Size & Age/Size Distributions

Measured by Employment (Business Dynamics Statistics: 1990-2006)

Pareto bounds, $[0.497, 0.937]$, and shape, (5.5), targeting unconditional size distribution

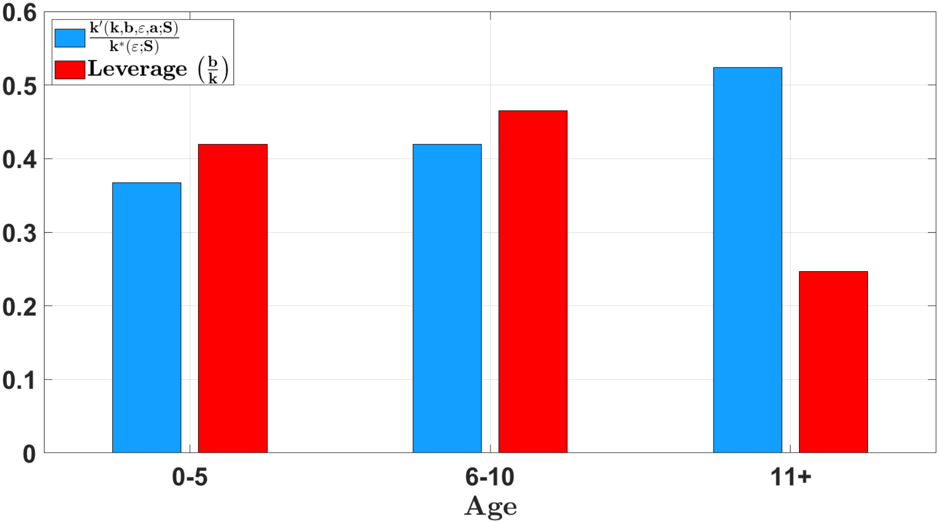
Employment Bins	Emp. Share	Pop. Share BDS	Pop. Share Model
Small (1-19)	0.201	0.885	0.880
Med. (20-499)	0.319	0.112	0.101
Large (500+)	0.480	0.003	0.019

Untargeted age-size distribution:

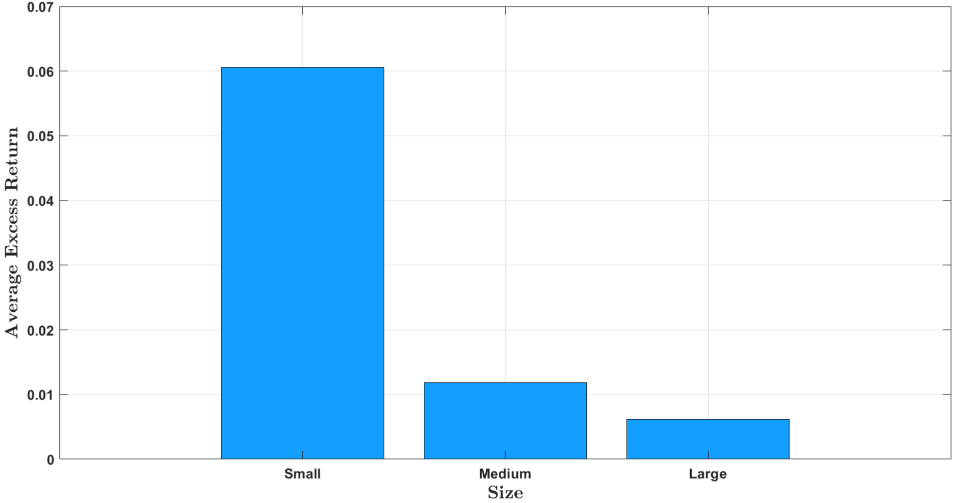
Age	0	1	2	3	4	5	Avg.
BDS:	0.260	0.338	0.378	0.417	0.451	0.477	0.368
Model:	0.260	0.302	0.362	0.427	0.506	0.595	0.377

Full Calibration back

Firm Life Cycle



Average Excess Return Across Firm Size



▸ Excess Return Expression

Debt Relief in a Crisis

33% drop in ζ, remains low for 4 periods, recovers 31.25%/year (Khan & Thomas, 2013)

- Supply shock in loanable funds market similar to 2008 (Duchin et al., 2010)
- 26% fall in borrowing in the model, matching fall in C&I loans 2008-2011

Common across all policies:

- Relief occurs on impact - policy pays fraction of outstanding debt
- Total size of policy held constant at 4% of steady state output
  - PPP loan forgiveness was roughly 3.7% of U.S. real GDP by 2023
- Taxes increase in period 7 to pay public debt
  - Half-life of output recovery
  - Gov pays 5% of its debt per year

Excess Return Policy

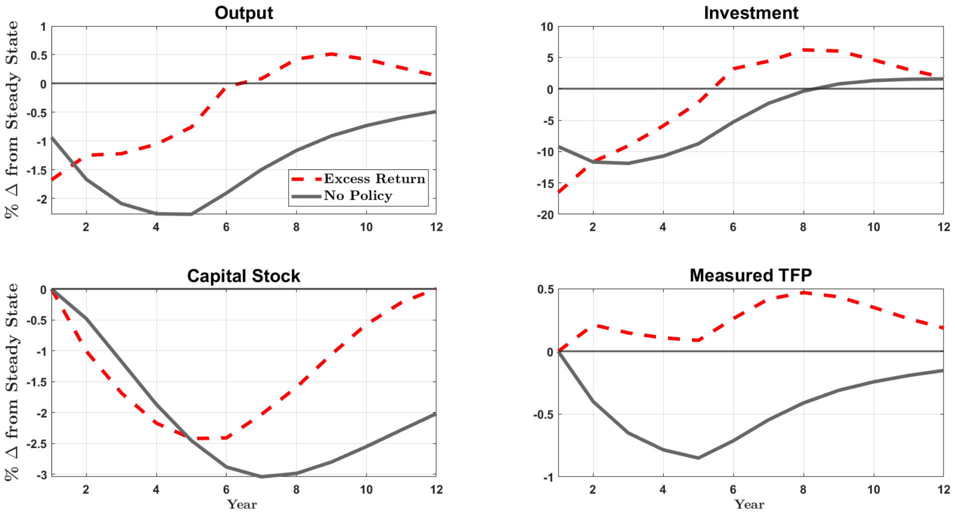
Reduce/equalize largest levels of excess return from top stair down

- Relieve debt of firm with greatest excess return to match 2nd greatest...
- Continue until policy funds exhausted

Does not relieve debt beyond what is necessary for investment

- All funds used for investment, not merely increasing share value
- Addresses concerns of debt-relief not reaching intended location (Li, 2021, Autor et al., 2022)

Policy Targeting Highest Excess Return Levels



▸ Additional series

Policy Targeting Small, Medium, Large firms

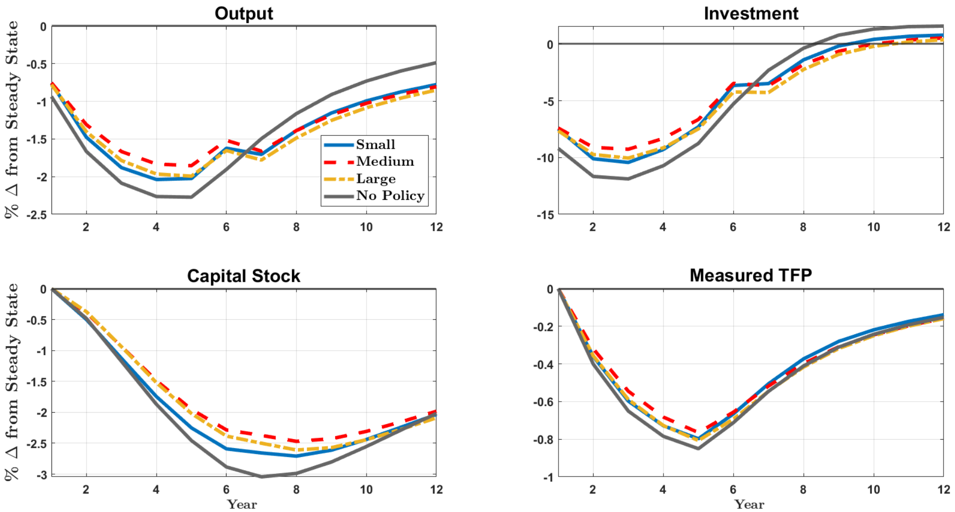
3 experiments for small, medium, large firm policy targets

- Smallest 88%, middle 10%, largest 2% of firms
- Eligibility based on model population shares Size Dist.

In order to keep total size of policy constant, relief per firm must vary across policies:

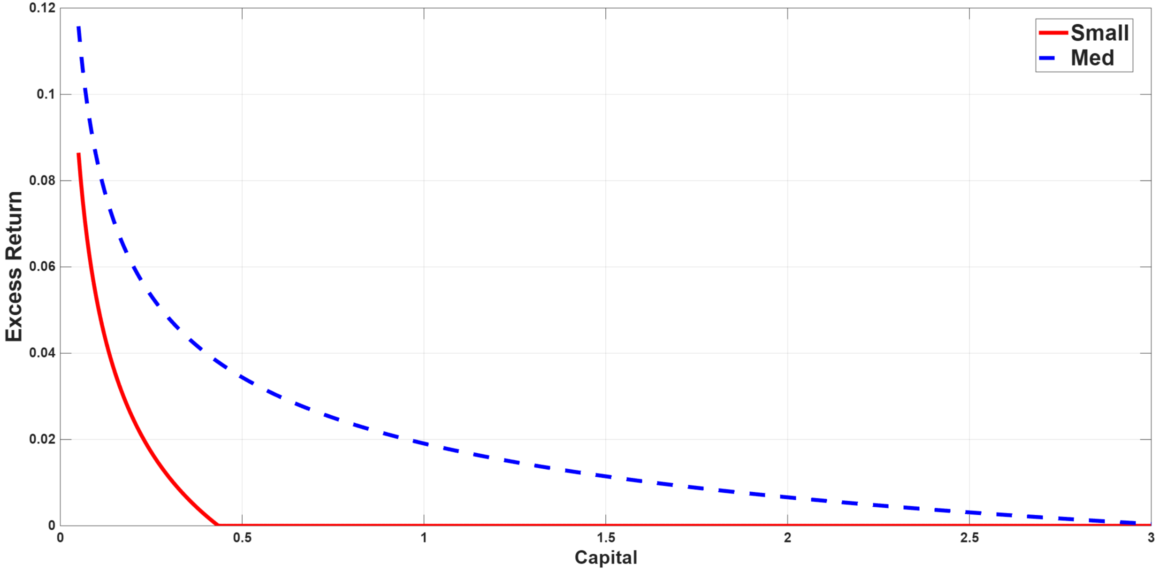
- Fraction of b paid for small firms: 0.237
- Fraction of b paid for medium firms: 0.145
- Fraction of b paid for large firms: 0.096

Policy Targeting Small, Medium, Large Firms



▸ Additional series

The Capacity for Growth



Conclusion

Non-linear model of heterogeneous firms and financial frictions, matching the unconditional size, and age-size, distributions of U.S. firms to study targeted debt relief in financial crisis

I consider policies targeting firms by excess return to investment, size, and age:

- Excess return policy outperforms all others
  - Fall in output diminished by 26% compared to no policy
- Among remaining policies, targeting medium size firms reduces fall in aggregates most
  - More likely to become large economic players than small firms
  - Further from their efficient investment than larger firms

Future work: alternative tax plan to mitigate dip, heterogeneity in new ε arrival probabilities

Appendix

Appendix

Policy Targeting Young, Middle Age, Mature Firms

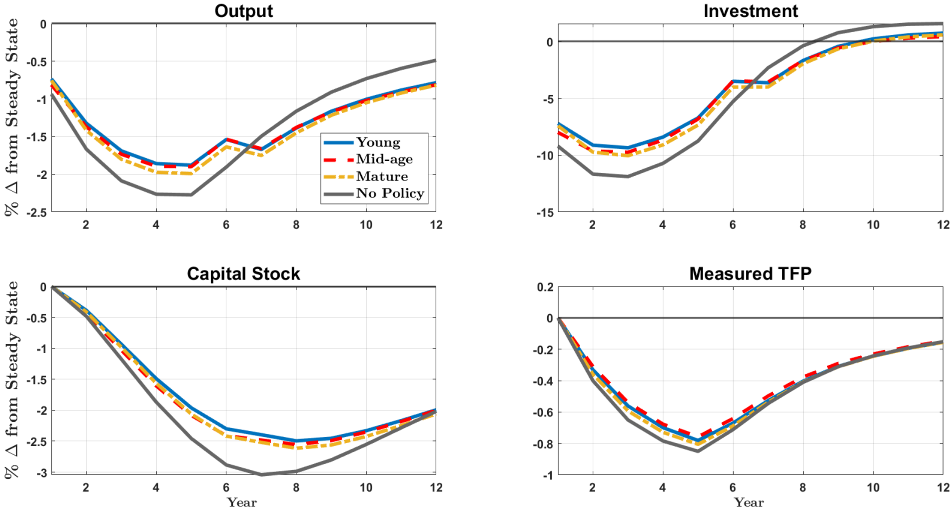
3 experiments for young, middle age, mature firm policy targets

- Age bins: [0 - 5], [6 - 10], 11+

In order to keep total size of policy constant, relief per firm must vary across policies:

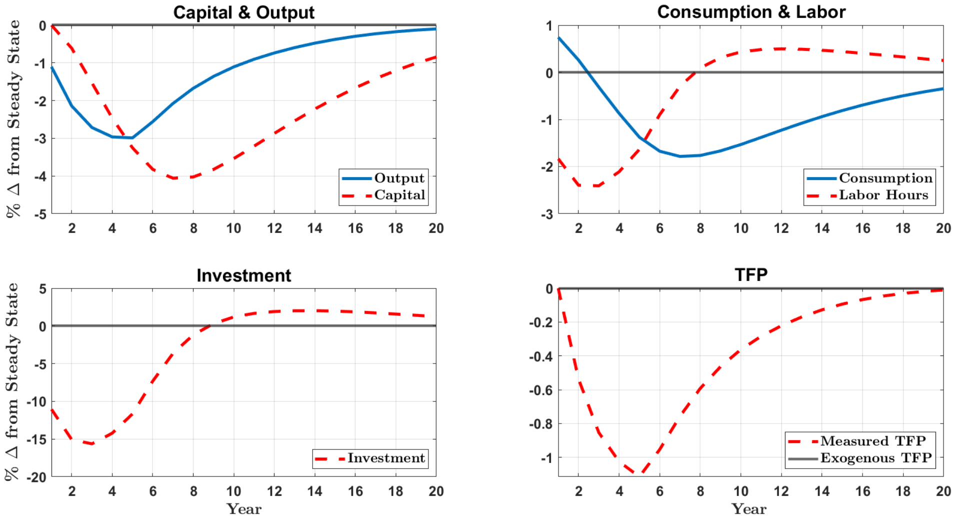
- Fraction of b paid for young firms: 0.352
- Fraction of b paid for middle age firms: 0.263
- Fraction of b paid for mature firms: 0.071

Policy Targeting Young, Middle Age, Mature Firms



▸ Additional series

Response to Credit Crisis: No Policy



Comparison to Untargeted Policy

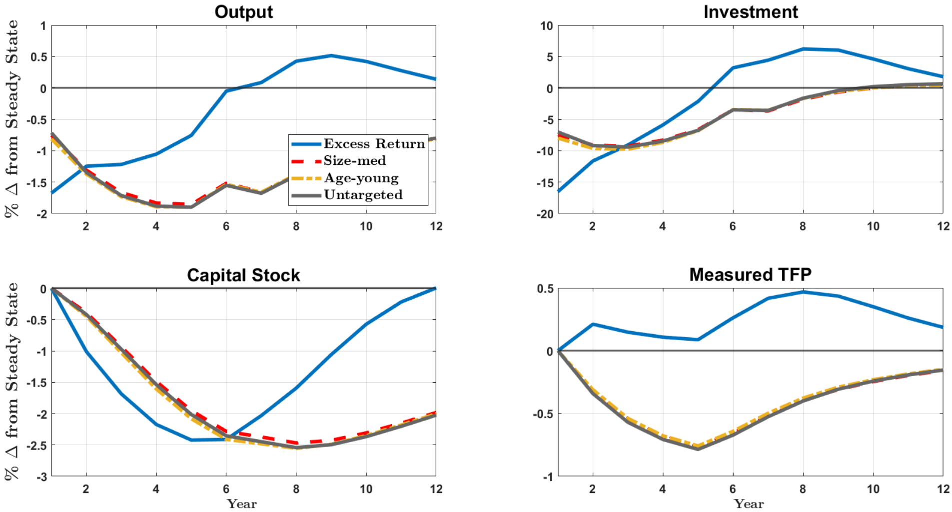
I consider one final policy: all indebted firms are eligible

- Fraction of total firm debt relieved: 0.047

Highlight effects of diminishing marginal returns

- Targeted policy can focus on key variables
- But, more concentrated policy $\rightarrow$ stronger diminishing marginal returns effects

Untargeted Policy Compared to Best Alternative Policies



Literature

- Fiscal policy to alleviate financial downturn:
  - Bianchi (2016), Jeanne, & Korinek (2020), Elenev, Landvoigt & Van Nieuwerburgh (2022), Angeletos, Collard, & Dellas (2023)
  - ▷ My contribution: Value of policy resources varies over nontrivial distribution of firms
- Distributional effects of policy intervention:
  - Guner, Ventura, Xu (2008), Buera, Moll, & Shin (2013), Gourio and Roys (2014), Jo & Senga (2019)
  - ▷ My contribution: Extend distributional analysis to the topic of debt relief in a crisis
- Transmission of shocks from financial conditions:
  - Kiyotaki & Moore (1997), Jermann & Quadrini (2012), Khan & Thomas (2013), Jo (2024)
  - ▷ My contribution: Analyze how debt relief may improve such conditions

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Household Problem

Representative HH maximizes lifetime value, $W(\lambda, \kappa; S)$

- Chooses: consumption, c, labor, n^h, shares, λ', bond holdings, κ'

$$W(\lambda, \kappa; S) = \max_{c, n^h, \lambda', \kappa'} \left[U(c, n^h) + \beta W(\lambda', \kappa'; S'_m) \right]$$

$$\text{st: } c + q(S_l)\kappa' + \int \rho_1(k', b', \varepsilon', a'; S'_m)\lambda'(d[k' \times b' \times \varepsilon' \times a') \leq$$
$$w(S)n^h + \kappa + \int \rho_0(k, b, \varepsilon, a; S_l)\lambda(d[k \times b \times \varepsilon \times a])$$

- ρ_1 is ex-dividend price of a share, ρ_0 is dividend-inclusive value of a share, $\Gamma(\mu)$ is known

back

Representative Household

- Period utility $U(C, 1-N)$, and discount factor $\beta \in (0,1)$
- Supplies: labor for wage $w(S)$, loans at risk free rate $q(S)^{-1}$
- Implied restrictions for equilibrium prices:
  - $w(S) = \frac{D_2 U(C,1-N)}{D_1 U(C,1-N)}$
  - $d_m(S_l) = \beta \frac{D_1 U\left(C'_m,1-N'_m\right)}{D_1 U(C,1-N)}$
  - $q(S_l) = \beta \sum_{m=1}^{N_s} \pi^s_{l,m} \frac{D_1 U(C'_m,1-N'_m)}{D_1 U(C,1-N)}$
  - Equilibrium decision rules $C = C(S)$, $N = N(S)$

HH Problem   Calibration

Market Clearing

$$\begin{aligned}
\textit{Capital}: \quad & K = \int k\mu(d[k \times b \times \varepsilon \times a]) \\
\textit{Labor}: \quad & N = \int n(k,\varepsilon)\mu(d[k \times b \times \varepsilon \times a]) \\
\textit{Output}: \quad & Y = \int z\varepsilon F(k, n(k,\varepsilon))\mu(d[k \times b \times \varepsilon \times a]) \\
\textit{Firm-debt}: \quad & B = \int (b|b > 0)\mu(d[k \times b \times \varepsilon \times a]) \\
\textit{Consumption}: \quad & C = Y - (1 - \pi_d(a))\left(K' - (1-\delta)\,k\right) \\
& \qquad + \pi_e\left((1-\delta)\,k - k_0\right)
\end{aligned}$$

Distribution

The distribution of firms is denoted by measure μ, defined on the Borel algebra, $\mathcal{S}$, generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E} \times \mathbf{A}$.

$\forall (A, \varepsilon_j) \in \mathcal{S}$ defines Γ, where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$

$$\mu' (A, \varepsilon_j) =$$

$$(1 - \pi_d(a)) \int_{\left\{(k,b,\varepsilon_i,a) | \left(g^K(k,b,\varepsilon_i,a;s,\mu), g^B(k,b,\varepsilon_i,a;s,\mu)\right) \in A\right\}} \pi_{ij} \mu \left(d \left[k \times b \times \varepsilon_i \times a\right]\right)$$

$$+ \pi_e \chi \left(k_0\right) H(\varepsilon_j)$$

Constrained and Unconstrained Investment

Collateral constraint binds to varying degrees across firms

Non-binding: $k'(k, b, \varepsilon, a; S) = k^*(\varepsilon; S)$

- Debt relief provides no extra investment
- Additional resources saved, $b' = \frac{k^*(\varepsilon,S)-x(k,b,\varepsilon,a;S)}{q(S)}$
  - Does offer protection from future binding constraint

Binding: $k'(k, b, \varepsilon, a; S) = x(k, b, \varepsilon, a) + q(S) \underbrace{\zeta k}_{b'=\zeta k} < k^*(\varepsilon; S)$

- Debt relief for these firms increases investment, reduces misallocation

Measuring Investment Inefficiency

Excess return to investment: expected discounted marginal value of investing minus cost

$$\mathbb{E}_{\pi^s}\,\mathbb{E}_{\pi^\varepsilon}\left[d_m(S_l)\left(\frac{\partial\pi(k',b',\varepsilon_j,a';S'_m)}{\partial k'}+(1-\delta)\right)\right]-1$$

where π is profit Parameterization

With efficient investment, excess return is 0

The further $k'(k,b,\varepsilon,a,S)$ is from $k^*(\varepsilon,S)$, the higher the return

Excess Return to Investment

Given the parameterization of the model, marginal benefit of capital investment should equal marginal cost of 1:

$$\frac{\alpha}{1-\nu}\frac{\beta}{p(S)}\left(\sum_{m=1}^{N_s}\pi^s_{l,m}p(S'_m)\sum_{j=1}^{N_\varepsilon}\pi^\varepsilon_{i,j}k'^{\frac{\alpha+\nu-1}{1-\nu}}\left[z_m\varepsilon_j\left(\frac{\nu z_m\varepsilon_j}{(1+\tau(S))w(S)}\right)^{\frac{\nu}{1-\nu}}\right.\right.$$
$$\left.\left.-(1+\tau(S))w(S)\left(\frac{\nu z_m\varepsilon_j}{(1+\tau(S))w(S)}\right)^{\frac{1}{1-\nu}}\right]+(1-\delta)\right)=1$$

where, $p(S)=\frac{\partial U(C,1-N)}{\partial C}$

With insufficient investment, LHS >1

back

Determination of Dividends

Final decision for continuing firms is their dividend payout

Constrained firms choose $D = 0$

- Greater value in investing those resources since excess return > 0

Unconstrained firms face one of two cases:

1. Positive probability of future binding collateral constraint:
   - Direct incentive to save: choose $D = 0$
2. No possibility of future binding collateral constraint:
   - *Minimum saving policy*; issue just enough dividends to remain unconstrained in all states

Min. Savings Sol.

Minimum Savings Policy

Define:

$$B(k,\varepsilon;S) \equiv \min_{\{\varepsilon_j|\pi_{ij}>0 \text{ and } S_m|\pi^S_{lm}>0\}} \widetilde{B}\left(k^*(\varepsilon),\varepsilon_j;S'_m\right)$$

where,

$$\begin{aligned}\widetilde{B}(k,\varepsilon;s,\mu) = z\varepsilon F\left(k,n^*(k,\varepsilon)\right) - \left(1+\tau(S)\right)w(S)n^*(k,\varepsilon) \\ + q(S)\min\left\{B(k,\varepsilon;S),\zeta k\right\} - \left(k^*(k,\varepsilon)-(1-\delta)k\right)\end{aligned}$$

and finally, dividends are:

$$D(k,b,\varepsilon,a;S) = x(k,b,\varepsilon;S) - k^*(k,\varepsilon) + q(S)\min\left\{B(k,\varepsilon;S),\zeta k\right\}$$

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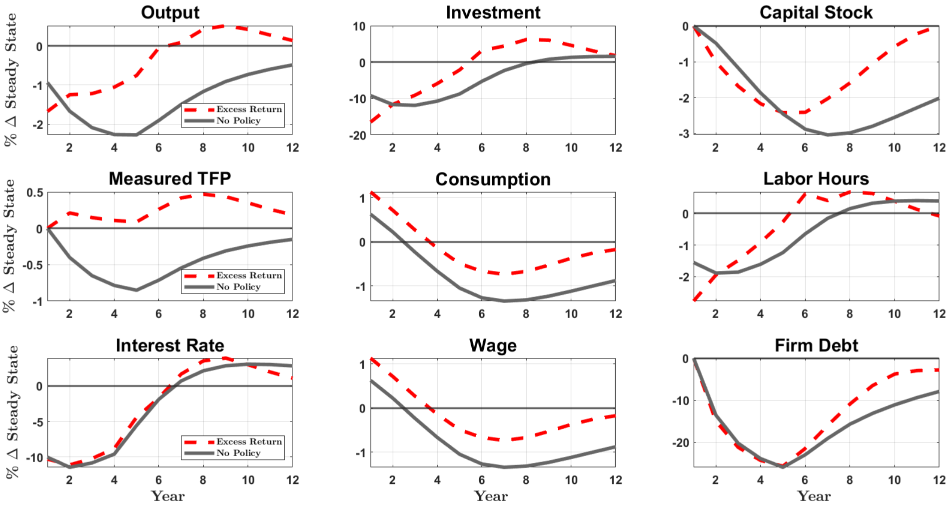
Extra Series - Excess Return Target



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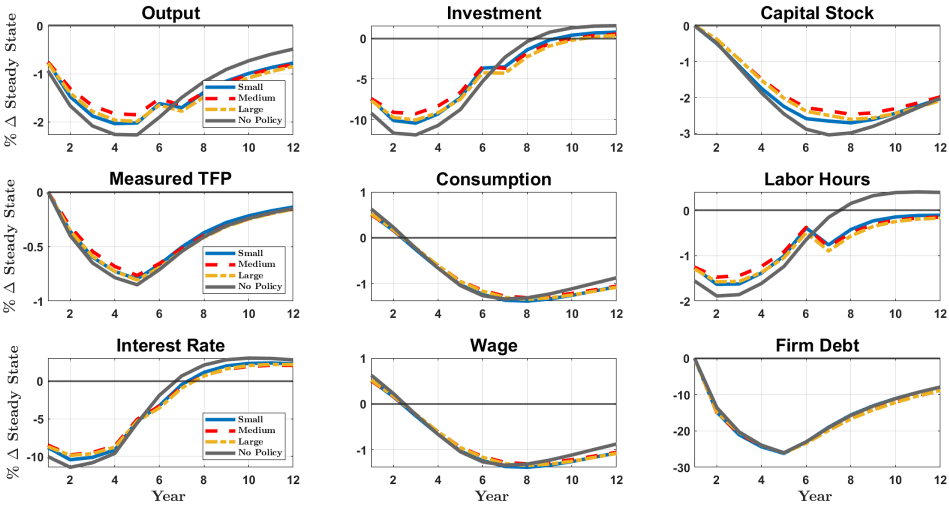
Extra Series - Size Targets



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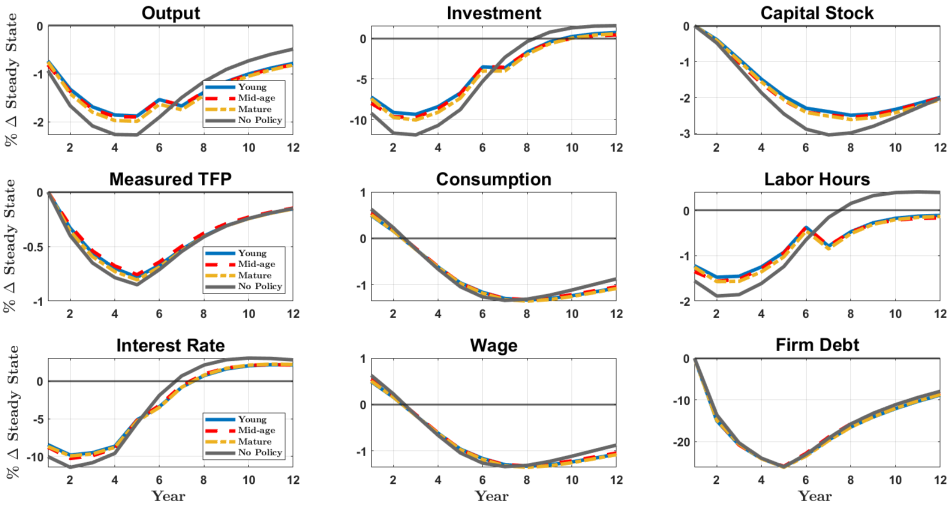
Extra Series - Age Targets



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Annual Calibration

$$U(C, 1-N) = \ln(C) + \psi(1-N) \qquad z\varepsilon F(k,n) = z\varepsilon k^{\alpha} n^{\nu}$$

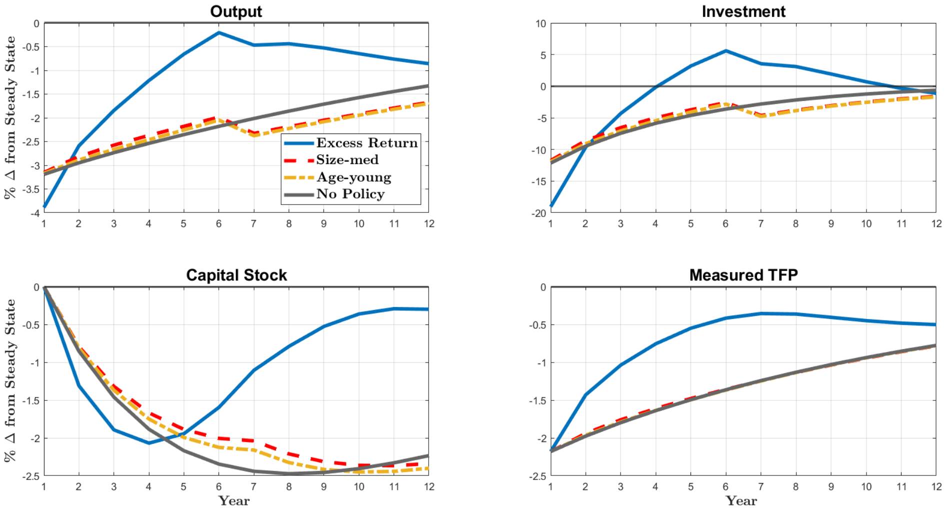
$$k_0 = \chi \int k\mu(d[k \times b \times \varepsilon \times a])$$

HH Conditions

Parameter			Target		Model
β	discount factor	$= 0.960$	real interest rate	$= 0.040$	0.041
ψ	leisure preference	$= 2.140$	labor hours	$= 0.333$	0.332
ν	labor share	$= 0.600$	labor share	$= 0.600$	0.600
δ	depreciation	$= 0.069$	investment/capital	$= 0.069$	0.069
$\frac{b_0}{k_0}$	entrant leverage	$= 0.400$	entrant leverage	$= 0.400$	0.400
α	capital share	$= 0.280$	capital/output	$= 2.300$	2.305
χ	fraction of entrant K	$= 0.208$	avg. n_0/N	$= 0.260$	0.260
ρ_ε	maintain ε	$= 0.990$	std dev. i/k	$= 0.337$	0.358
ζ_0	collateral fraction	$= 0.981$	debt/assets	$= 0.372$	0.372

back Exit Rates

Selected Policies Following TFP Shock



- 2% shock with persistence of 0.909

Firm Exit Rates by Age (BDS, 1990-2006)

Age:	1	2	3	4	5	6-10	11+
$\pi_d(a)$:	0.2478	0.1640	0.1356	0.1174	0.1062	0.0840	0.0655

Firm entrant rate $= 10.6\%$

- Entrant rate selected to keep mass of firms at 1

▸ back

Perfect Foresight Solution -1

1. Guess a vector of $\{\hat{\tau}\}_0^{T+1}$ and $\{\hat{C}\}_0^{T+1}$
   - τ' is needed for k' decision
   - $\hat{C}$ implies a $w(S)$ and $q(S)$
2. Back-solve decision rules from date T
3. Forward-solve the distribution (and find aggregates) for each T
4. Back out $\tilde{C}$ implied by aggregate resource constraint
5. Solve for $\tilde{\tau}$ by following:

Determining $\tau(S)$

Define: $Balance \equiv \hat{\tau}(S)w(S)N(S) + q(S)\theta' - \theta - T(\Theta, S)$

Then, $\hat{\tau}(S)w(S)N(S) = Balance + q(S)\theta' - \theta - T(\Theta, S)$

Define: $\Delta\hat{\tau}(S)$ as change in $\hat{\tau}(S)$ such that:

$$(\hat{\tau}(S) + \Delta\hat{\tau}(S))w(S)N(S) = \theta + T(\Theta, S) - q(S)\theta' \quad \underline{and} \quad \theta' = (1-\phi)\theta$$

- This is the increase in $\hat{\tau}(S)$ needed to set $Balance = 0$, while the fiscal rule is satisfied

Then: $\Delta\hat{\tau}(S) = \left((\theta + T(\Theta, S) - q(S)\theta')\left(\frac{1}{w(S)N(S)}\right)\right) - \hat{\tau}(S)$

So, $\tilde{\tau} = (\hat{\tau}(S) + \Delta\hat{\tau}(S))$

6. Check guess, set $\{\tilde{\tau}\}_0^{T+1} = \{\hat{\tau}\}_0^{T+1}$ and $\{\tilde{C}\}_0^{T+1} = \{\hat{C}\}_0^{T+1}$, and repeat govcrisis

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